Finite Element – Integral Equation Full-Wave Multi-Solver for Efficient Modeling of Resonant Wireless Power Transfer

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In resonant WPT (wireless power transfer) systems several components with different material properties are often in the vicinity of the resonant coils. In order to efficiently model such systems a novel full-wave multi-solver is developed where the TVFEM (tangentially continuous vector finite element method) is coupled with a MoM (method of moments) solver based on an integro-differential system specifically designed to model the near singular behavior of the thin coil wires. A simplified sequential version of the multi-solver is also introduced where the electromagnetic field of the coils in homogeneous material background is considered as an incident field and thus computed by the MoM technique as a first step. The incident field then plays the role of the source for the TVFEM utilizing the scattered field formulation, thereby calculating the scattered part of the field as a second step. Numerical results are presented to illustrate the behavior of the multi-solver approach.

***Index Terms*—finite element method, integral equation, multi-solver, resonant wireless power transfer.**

I. Background

COMPUTATIONAL electromagnetics is a critical tool in the R&D process of resonant WPT (wireless power transfer) systems [1]. Most modern WPT systems are based on the inductive coupling of resonant coils in the near field. This seems to justify their – often component-wise – static or quasi-static electromagnetic analysis as common practice. However, full-wave modeling of the whole system is proven to provide better prediction for the conditions of optimal power transfer – a crucial knowledge for WPT – than the quasistatic approach [1]. In addition, the full-wave analysis accounts for the radiating phenomenon, which is important in the investigation of electromagnetic compatibility (EMC) and human exposure aspects of WPT [2].

Recently we proposed a lightweight computational method to simulate and optimize the whole WPT system provided the surrounding medium is homogeneous [3]. This fast method with a low-memory footprint is a MoM (method of moments) technique based on a set of integro-differential equations specifically designed to deal with the near-singular behavior of the coil wires. In some important applications, for example, in vehicle charging systems and medical implants, the environment is typically heterogeneous, that is, objects with different material properties are in the vicinity of the resonator coils. In this case the integral equation approach becomes quite inefficient. Therefore one usually utilizes FE (finite element) computational models that can inherently handle material inhomogeneities. However, in order to accurately model the thin wires of the coils by the FE technique, very fine discretization is necessary at the price of very high computational costs.

Several attempts have already been made to reduce the computational burden of the FE method in the context of WPT modeling, like the domain decomposition [4] or the homogenization techniques [5]. We follow a different path here and develop a new multi-solver that couples our integral equation technique [3] and our TVFE (tangentially continuous vector finite element) implementation [6] in an overlapping scheme.

II. Theory

A. Field Components

Figure 1 shows an illustrative WPT application in heterogeneous environment. In this situation the electromagnetic field can be split into two components: one is the incident field, that is, the coil field in the infinite homogeneous background (usually air); the other is the scattered field induced by the presence of different objects with complex relative material parameters ε_r and ν_r. Thus, the electric field can be written as

$$\vec{E} = \vec{E}^I + \vec{E}^S \tag{1}$$

All other field quantities are split similarly. Based on such splitting we can introduce efficient numerical formulations in which the incident and scattered fields are computed by different numerical techniques, thereby maximizing efficiency.

B. Integral Equation Method

We define the coordinate ς as the position along the curve of the coil in Figure 1. The points $\varsigma = 0$ and $\varsigma = \ell$ correspond to the beginning and the end of the wire. Then, the incident field can be expressed very efficiently by the integral equation technique introduced in [3]. The governing integro-differential system is

$$A_\varsigma^I = \hat{e}_\varsigma \cdot \frac{\mu_0}{4\pi} \int_0^\ell \frac{e^{-jk_b D(\varsigma,\varsigma')}}{D(\varsigma,\varsigma')} I(\varsigma') d\overline{\varsigma}' \tag{2}$$

$$\Phi^I = \frac{1}{4\pi\varepsilon_0} \int_0^\ell \frac{e^{-jk_b D(\varsigma,\varsigma')}}{D(\varsigma,\varsigma')} q(\varsigma') d\varsigma' \tag{3}$$

$$rI(\varsigma) = E_\varsigma^I = -d\Phi^I / d\varsigma - j\omega A_\varsigma^I \tag{4}$$

$$j\omega q(\varsigma) + dI(\varsigma)/d\varsigma = 0 \tag{5}$$

which is solved for unknowns line charge density q, current I, ς component of vector potential A_ς, and electric scalar potential Φ as described in [3]. In (2)-(5), $k_b = \omega\sqrt{\varepsilon_b \mu_b}$ is the wave number of the background material, $D(\varsigma, \varsigma')$ is the distance between points ς and ς', and r denotes the resistance per unit length of the coil wire. The electromagnetic field is excited by prescribing the current values at the coil terminals.

C. Coupling by Scattered Field Method

This approximation assumes that the material inhomogeneity does not affect significantly the distribution of q and I in (2)-(5) compared to the homogeneous situation. Thus, the electromagnetic field of the coils in homogeneous background can be considered as incident field and computed in the first step. Then, the scattered component of the field due to the presence of material inhomogeneity can be determined by a TVFEM implementation as described in [6]. Following [6] the wave equation for the scattered field is

$$\nabla \times \nu_r \nabla \times \vec{E}^S - k_0^2 \varepsilon_r \vec{E}^S = k_0^2 \left(\varepsilon_r - \frac{\varepsilon_{rb}\nu_r}{\nu_{rb}} \right) \vec{E}^I + j\omega\mu_0 \nabla \frac{\nu_r}{\nu_{rb}} \times \vec{H}^I \tag{6}$$

where ε_{rb} and ν_{rb} are the relative background material parameters, and k_0 is the vacuum wave number. Note, that while deriving (6) we make use of the fact that the incident field satisfies the vector wave equation with background material parameters. The magnetoquasistatic version of the scattered field method is discussed in [7].

Once the incident and scattered components are available the impedances can be calculated as described in [3] by using the total field values as defined in (1).

D. Fully Coupled Multi-Solver

We can construct a more accurate computational model by solving the incident problem (2)-(5) and the scattered problem (6) simultaneously. In order to build the novel coupled system we need to append system (2)-(5) by the scattered field components. For example in (4) we consider E_ς instead of E_ς^I. Similarly the incident field components in (6) become unknowns.

The resulting linear system has a sparse block corresponding to the FE unknowns and a significantly smaller dense block corresponding to the integral-equation unknowns. An efficient iterative solver is developed to retain most of the advantages of a sparse system. The detailed description of the iterative solver together with the comparison of the two solution approaches will be discussed in detail in the full version.

The verification of the multi-solver is performed by comparing results to full FE solutions where a significant improvement of the run-time performance is also observed.

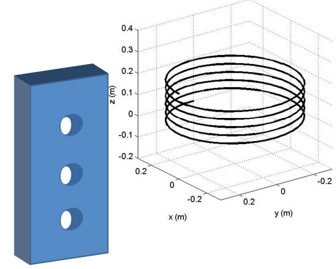

Fig. 1. An illustrative heterogeneous WPT arrangement

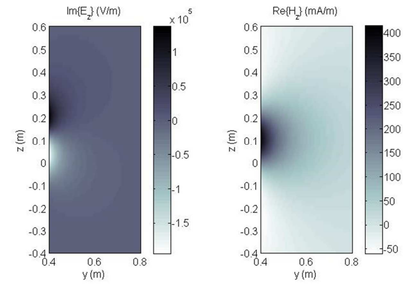

Fig. 2. The *z* component of the electric and magnetic fields of the coil on the x=0 plane

III. Results

Figure 1 shows a WPT arrangement where the coil is close to an object with material properties different from the background. The helical coil with radius 30 cm and height 20 cm has 5.25 turns. The radius of the copper wire is 3 mm. The coil is driven by a 10.3 MHz sinusoidal current whose amplitude at the terminals is prescribed to 1 mA. The 10.3 MHz is the first self-resonant frequency of the coil so the maximal amplitude of the current along the wire is much higher than at the terminals; it peaks at around 100 mA. Figure 2 depicts the *z* component of electric and magnetic fields of the coil at the material inhomogeneities.

References

[1] Sz. Gyimóthy, S. Bilicz, J. Pávó, L. Tóth, G. Varga, and L. Szűcs, "Field Computational Aspects of Wireless Power Transfer," *IGTE Symposium*, Graz, Austria, Sept 2014, no. I3-1.

[2] Ping-Ping Ding, L. Bernard, L. Pichon, A. Razek, "Evaluation of Electromagnetic Fields in Human Body Exposed to Wireless Inductive Charging System," *IEEE Trans. Magn.*, vol.50, no.2, pp.1037, 2014.

[3] S. Bilicz, "High-frequency modelling of coils by integral formulations," *IGTE Symposium*, Graz, Austria, Sept 2014, no. P1–12.

[4] M. Benhamouche, L. Bernard, M. Ibrahim, L. Pichon, "Domain decomposition method for solving complex magnetodynamic problem: application to wireless charging system," *IEEE CEFC*, Annecy, France, May 2014, no. PB2-2.

[5] D.W. Ferreira, R.V. Sabariego, L. Lebensztajn, L. Krahenbuhl, F. Morel, C. Vollaire, "Homogenization Methods in Simulations of Transcutaneous Energy Transmitters," *IEEE Trans. Magn.*, vol.50, no.2, pp.1017-1020, Feb. 2014.

[6] I. Bárdi, Z. Badics and Z. J. Cendes, "Total and Scattered Field Formulations in the Transfinite Element Method," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 778-781, 2008.

[7] Z. Badics *et al*, "An effective 3-D finite element scheme for computing electromagnetic field distortions due to defects in eddy-current nondestructive evaluation," *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 1012-1020, 1997.